Constructing The Cheops Pyramid Through Multispiral Ramps

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Pre-release, introduction only

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1. About this Work

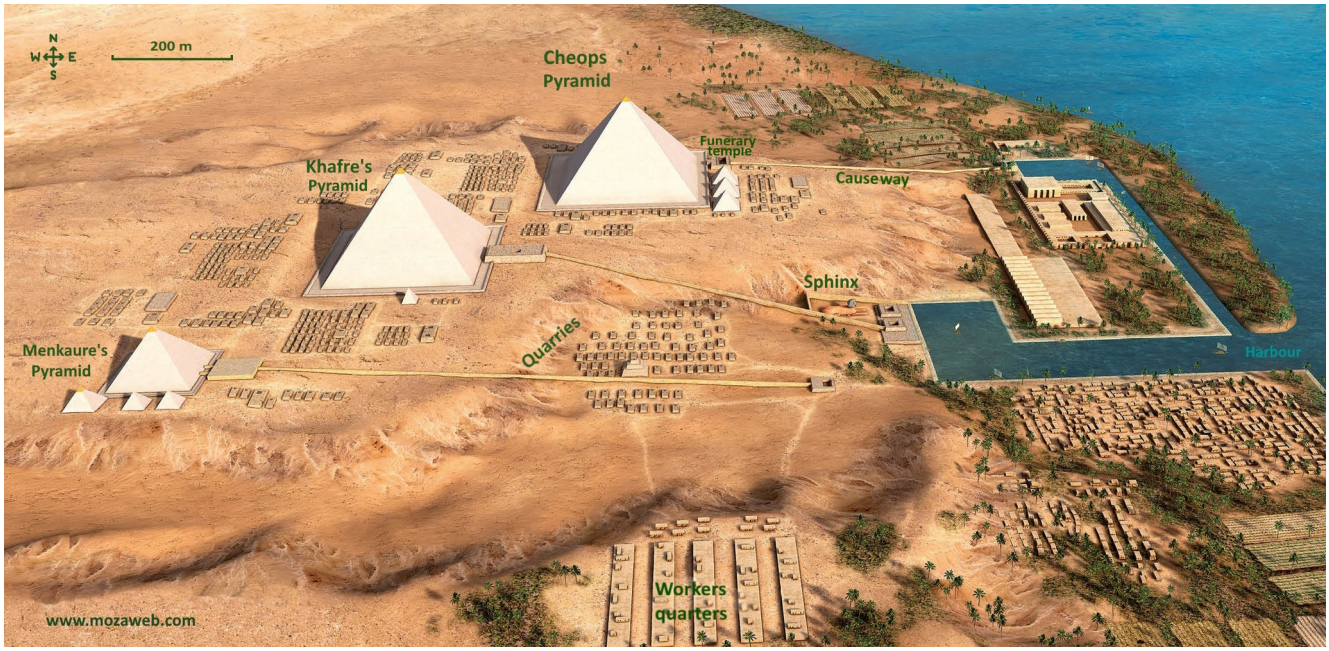

Image 1 - The Pyramids of Giza [2]

The Cheops Pyramid is considered as the tomb of pharaoh Cheops or with Egyptian name Khufu. It was built in the 26th century BC is the oldest of the three pyramids of Giza.
It is the largest Egyptian Pyramid with a weight of almost 7 million tons and therefore is also noted as the Great Pyramid. [3]

The Cheops Pyramid is the oldest of the Seven Wonders of the Ancient World and it is the only one to remain largely intact. [4] [5] [6]

Even though the Great Pyramid is one of the most impressive and tallest man-made structures, there are almost no documents or archaeological finds that indicate the construction technique. How did the Egyptians in the Old Kingdom manage to build this pyramid? [3]

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1. https://scholarship.claremont.edu/jhm/
2. With kind permission of Mozaik Education Ltd.
   https://www.mozaweb.com/en/search?search=egyptian+pyramids&view=grid&sort=grouped
3. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza
4. https://en.wikipedia.org/wiki/Seven_Wonders_of_the_Ancient_World
5. Mark Lehner and Zahi Hawass, Giza and the Pyramids, The University of Chicago Press, 2017, Side 143
6. https://www.pyramid-of-giza.com/the-great-pyramid-of-giza-pyramid-of-khufu/

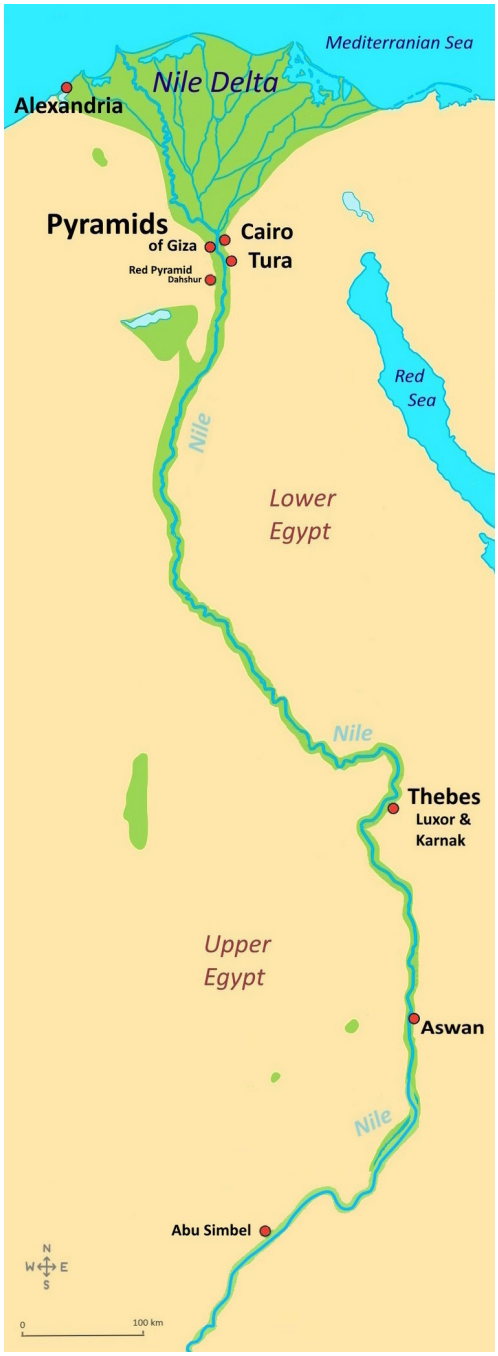


Image 2 - Map of Egypt [7]

The first author to mention the pyramids was Herodotus, but that was around 450 BC, more than 2000 years after the monument was built.
He received his information from Egyptian priests and some of his writings do not seem to be very reliable. These statements can be regarded as very probable: [8] [9] [10]

a1) The construction time of the Cheops Pyramid was 20 years.
a2) In the first ten years a wide causeway was erected, what measured nearly 1 kilometer and 18.3 m wide. It consisted of stone polished and carved with figures.
a3) In order to reduce sliding friction, the working paths were partly made of smoothed limestone.
a4) Completion of the outer casing was from the top to the base of the pyramid.
a5) Labourers worked in 3-month shifts.

Diodorus did visit Egypt around 60 BC and his writings seem to be more reliable: [9]

b1) Diodorus agrees with Herodotus on the construction time of 20 years.
b2) The pyramid was built with the help of ramps.
b3) The ramps were removed after the pyramid was completed.
b4) Lifting tools had not yet been invented when the pyramids were built.
b5) The casing of the pyramid was still in excellent condition at his visit, whereas the uppermost part of the pyramid was formed by a platform 3.1 m wide.
b6) He estimated the number of workers at 360 thousand.
b7) Cheops was not buried within the pyramid, but rather in a secret place.

Up to now, many attempts have been made to explain the **transportation problem**.
How could such a huge mass of stones be transported and positioned within such a short time?

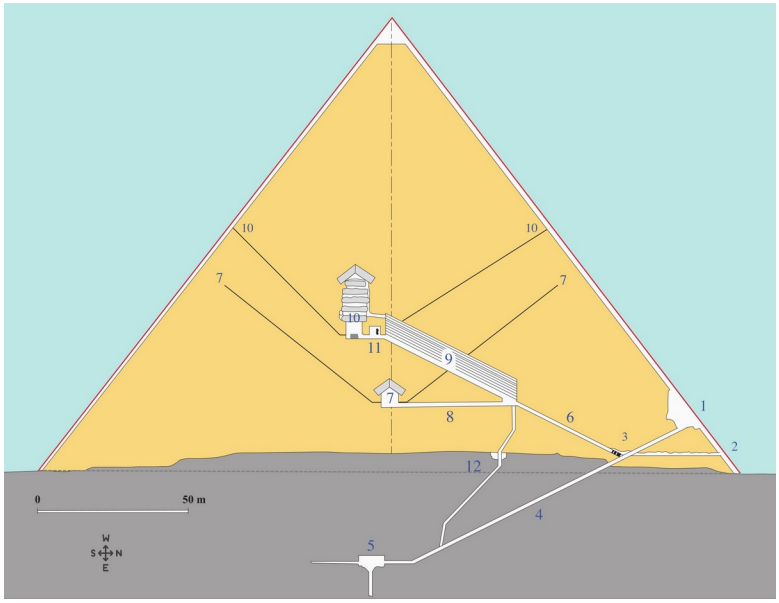
Many construction methods have been proposed, but so far none of them has solved the problem in a satisfactory way. [10] 11] [12]

That is why there is plenty of room for speculations. It even goes so far as claiming that the pyramids were built by aliens, what was publicly reiterated by one of the richest men in 2020. [13] [14]

7. https://de.123rf.com/ #30823347, modified map
8. https://en.wikipedia.org/wiki/Herodotus#Author_and_orator
9. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza
10. Frank Müller-Römer, Der Bau der Pyramiden im Alten Ägypten, Herbert Utz Verlag, 2011, Chapter 7
11. Frank Müller-Römer, Pyramidenbau mit Rampen und Seilwinden, 2007 https://edoc.ub.uni-muenchen.de/8064/1/mueller-roemer_frank.pdf
12. https://cheops-pyramide.ch/pyramid-building.html
13. https://en.wikipedia.org/wiki/Pyramidology
14. https://www.stern.de/panorama/wissen/elon-musk-verbreitet-irre-pyramiden-these-und-wird-von-aegyptischer-regierung-zurechtgewiesen--9361356.html

The pyramid has the following properties: [15] [16]

c1) Length of 230.33 m on each side.
c2) Height of 146.59 m.
c3) There are 210 platforms of about 2.3 million blocks of stone.
c4) About 130 blocks of granite up to 80 tons were transported to a height of 70 m.
c5) About 9 blocks up to 400 tons were transported to a height of 85 m.
c6) The construction was done with very high precision.
c7) Almost no remainders were found outside the pyramid.
c8) The pyramid got almost no damages caused by three severe earthquakes.



1. Original entrance, North Face Corridor
2. Robbers' Tunnel, tourist entrance
3, 4. Descending Passage
5. Subterranean Chamber
6. Ascending Passage
7. Queen's Chamber & its “air-shafts”
8. Horizontal Passage
9. Grand Gallery
10. King's Chamber & its “air-shafts”
11. Corridor to the Sarcophagus Chamber and the Blocking Stone Chamber
12. Grotto & Well Shaft

Image 3 - Elevation diagram viewed from the east side [17]

The first and major proposal was to create a straight front ramp. Assuming an angle of slope with 6.6 degrees, this would have a length of 1.2 km and a volume of 135% based on the pyramid itself. Since the two neighboring pyramids did not yet exist at that time, the ramp could have been arranged as shown in Image 4. This is probably the only possibility to place such a ramp in this landscape. It would take more than 10 years to remove this ramp, and therefore, it is impossible to finish the monument within 20 years. If there would not be the crucial time limit, then since long, the straight front ramp would be considered as the solution to the problem.

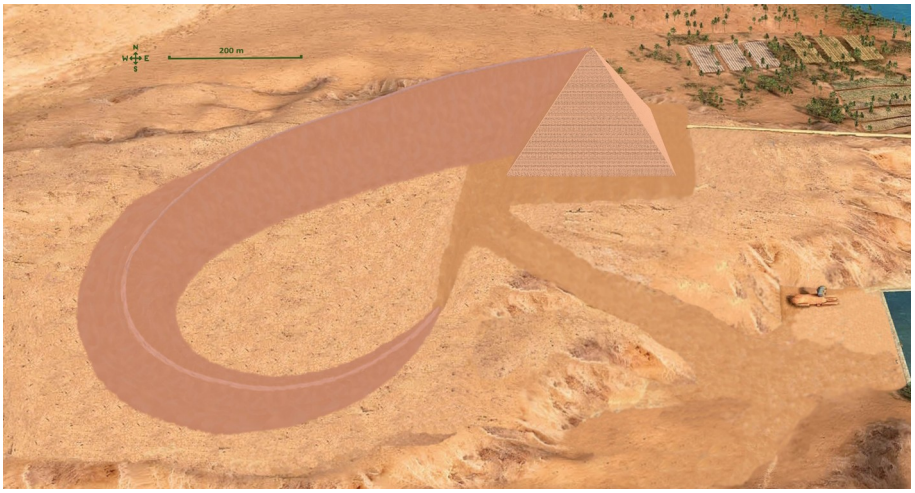


Image 4 - Straight Front Ramp [18]

15. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza
16. https://en.wikiarquitectura.com/building/great-pyramid-of-cheops/
17. https://de.wikipedia.org/wiki/Datei:Cheops-Pyramid.svg
18. Background element from https://www.mozaweb.com

This work aims to reinvent a possible construction method of the pyramid. This is not done by just indicating a method, but through a detailed construction planning. Only those techniques are considered which had been customary at that time.

These are the essential requirements and assumptions for the project:

d1) The ramps were made of dried bricks.
d2) The angle of the ramps was 6.6 degrees.
d3) The stone blocks were transported on wooden rollers.
d4) Transport of the blocks with workers and not with oxen.
d5) Transport speed was 3 m/min.
Thus, for each delivery lane one block could be transported every 4 minutes.
d6) The workers of a team pulled all at once for 1 second with a force of 40 kp each.
Thus, they covered a distance of 25 cm. Then they had 4 seconds for preparing the next pull.
d7) There were 20 thousand workers for 3 months during the Nile flood,
and only 10 thousand workers for the rest of the time.
d8) It took 8 labourers to work for one day to produce and smooth a block.
d9) A working day consisted of two shifts of 6 hours each.
d10) There were 300 working days per year.

These are the main results:

e1) The main ramp shown in green color leads to the top of the pyramid.
e2) The other 6 ramps end at different heights. The ends are marked with red color.
e3) The ramp volume requires only 8.4% of material based on the pyramid volume.
The many individual paths require much less material than a unique broad path.
e4) A pyramidion 3.25 m wide, 2.1 m heigh and weighting 20 tons could be set up without any problems.
e5) The casing was installed from top to bottom by removing the corresponding ramps.
e6) The construction time of the pyramid was almost exactly 20 years.
e7) Constructing the pyramid required 250 thousand working years.
e8) The requirements and results are consistent with the reports of Herodotus a1) – a5) and Diodorus b1) – b7).

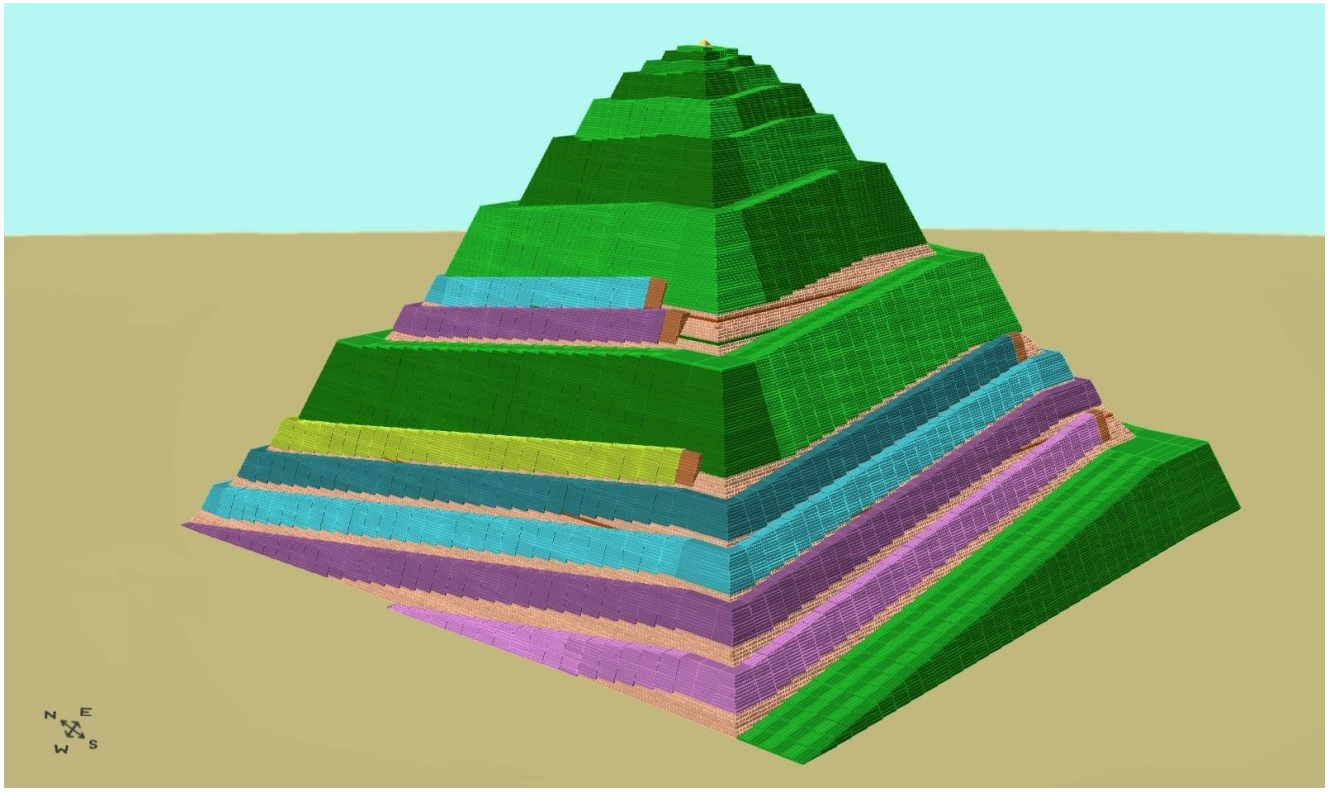


Image 5 - Multispiral Ramps seen from the south-west side

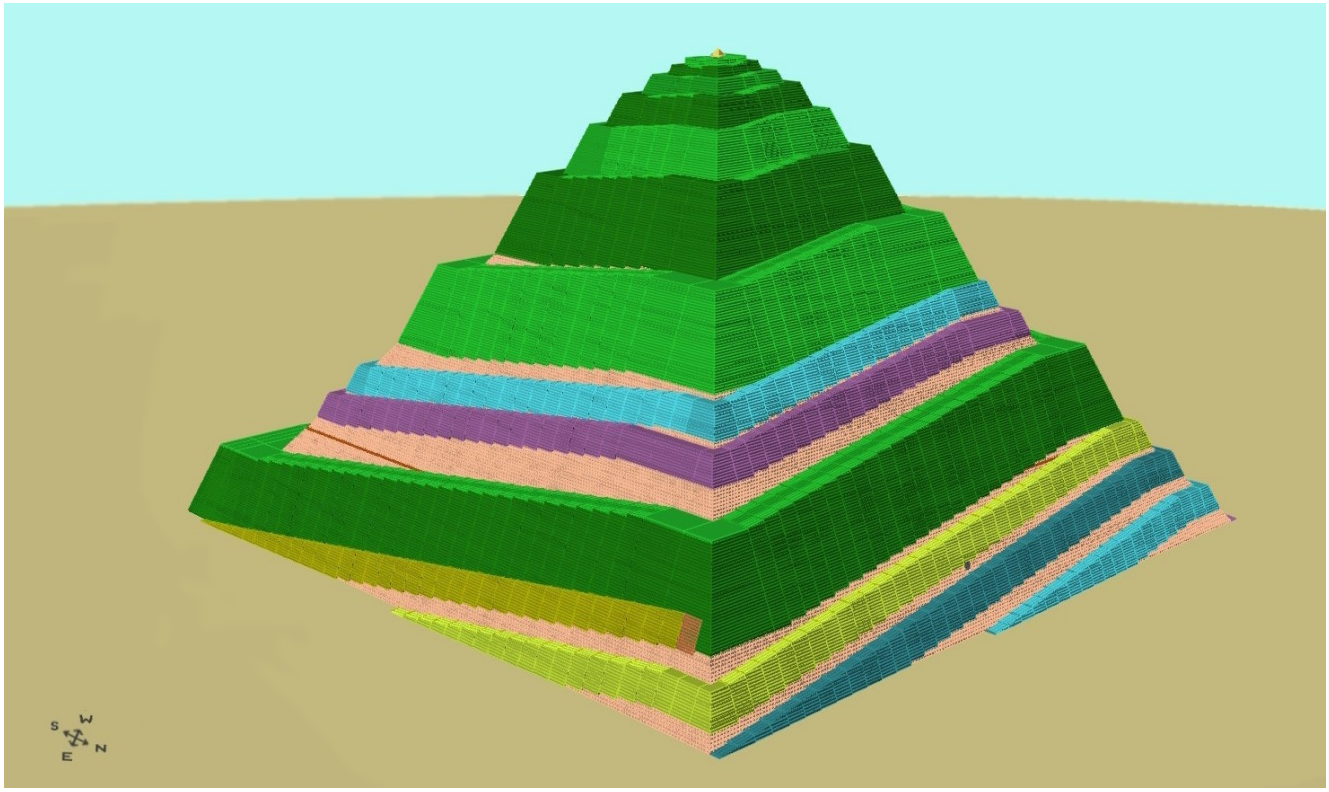


Image 6 - Multispiral Ramps seen from the north-east side [19]

The green main path leads till the top of the pyramid. With each rotation, the color changes between light and dark to show the 8.5 rotations. On 3 sides there is the beginning of 2 small individual paths with one lane each. All paths are shown in different colors so it is easier to follow their course.
The object resembles a square party hat with a jewel on top because of the many colors. The real appearance, however, was less impressing, since the desert sand, the limestones, and the bricks of the ramps all had a similar yellowish-brown color.

Exact calculations can be done based on the detailed planning model:

- f1) The number of blocks can be determined for a specific level.
- f2) The model can be used to determine how many delivery lanes lead to this level. A total of 180 blocks can be delivered per lane and working day.
- f3) The number of tow teams that are on a delivery lane can be determined. The tow teams are 12.5 m apart, what corresponds to a height difference of 2 levels.
- f4) The available workers are distributed into the areas towing and producing.
- f5) The number of blocks delivered per working day can be evaluated by summing up the values of all delivery lanes.
- f6) The exact construction time for each level can be evaluated.
- f7) The total construction time of the project can be determined by summing up the corresponding values of all 210 levels.

The idea of positioning ramps parallel to the pyramid was first mentioned by Uvo Hölscher in 1912 and by E. Landt in 1923. [20] [21]
In 1977 Georges Goyon published his proposal with a single spiral ramp, the so-called the wrap-around method. This is the most discussed proposal so far.

19. All three-dimensional CAD models were created by Rudolf Höld
20. Uvo Hölscher, Das Grabdenkmal des Königs Chephren, Veröffentlichungen der Ernst von Sieglin Expedition in Ägypten *I.* Leipzig, 1912, https://gizamedia.rc.fas.harvard.edu/documents/holscher_chephren.pdf
21. E. Landt, Ein neuer Kampf um die Cheops-Pyramide, Weidmannsche Buchhandlung, Berlin, 1923

In 1998 Klemm and Klemm submitted their proposal for the integral ramp. There are two different ramp paths that start on opposite sides of the pyramid. [22]

The basic idea of the theory of Multispiral Ramps was already published by Nikolaus Willburger in 2002. The term “function ramp” is used in this work. The placement of the pyramidion is described in detail, but the construction of the lower part of the pyramid is only indicated. According to the title “Theoretical Considerations”, the detailed elaborations and calculations are missing. [23]
This work contains the basic ideas for solving the problem and was cited as a “purposeful approach” in the central book on the construction of the pyramids. [24]
Therefore, it is quite surprising why this work has received only little attention within 20 years and no one has tried to develop it further.

This work was developed independently. On October 17th, 2021, Volz watched a video about “the unsolved mysteries of the Egyptian pyramids”. This was the impetus for the project. [25]
Thus, the date of publication corresponds to the 30th month since the start of the project.

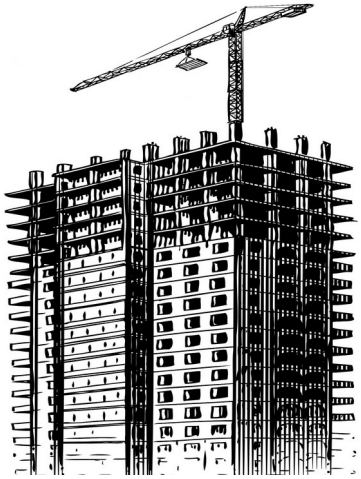


Image 7 - Climbing crane [26]

The decisive information from this video was the large volume of ramp material, which could not have been removed in the specified construction time of 20 years.

Therefore, a method had to be found to minimize the ramp material.

Right away there was the idea of a climbing crane. This does not stand on its own tower, but is placed on top of the existing building and grows with it.

This means for the pyramid, that the ramps must be placed on the already built and truncated pyramid.

Image 8 - Model with 400 blocks

Then a small model with 400 building blocks was made. After trying back and forth, it quickly became clear how to place the ramps on the step pyramid and how to simulate the construction process.
It was even possible to set up the pyramidion at the end of the bottom-up process.
In the subsequent top-down process, the white casing stones were positioned and the ramps were removed in parallel.

So far so good. The basic principle of construction was discovered, but calculations were not possible at this state. Although the blocks are only 2 cm wide, a complete model would have a base of almost 4.6 m and would require 3 million blocks.

22. Frank Müller-Römer, Der Bau der Pyramiden im Alten Ägypten, Herbert Utz Verlag, 2011, section 7.4.3
23. Nikolaus Willburger, Die Technik des Pyramidenbaus zu GIZA, Theoretische Überlegungen zu „Funktionsrampen“ beim Bau der Pyramiden, 2002. https://www.archaeologie-online.de/artikel/2002/die-technik-des-pyramidenbaus-von-giza/
24. Frank Müller-Römer, Der Bau der Pyramiden im Alten Ägypten, Herbert Utz Verlag, 2011, section 7.4.9
25. https://www.youtube.com/watch?v=03mJcLCSaVM
26. Alamy, HNPW42

The first crucial and fundamental step happened unexpectedly through the calculation of the volume of parallel ramps:

g1) The volume of a parallel ramp increases quadratically with the width of the path. This can easily be seen by a mathematician, but for a non-mathematician it may be confusing at first. Simply expressed, if you double the width of a path, then the ramp material is not doubled, but quadrupled.

So far it was not possible to find this fundamental result in the literature.
In order to minimize the ramp material, a ramp model must consist of as many individual paths with minimal width. This is the fundamental idea of the theory of Multispiral Ramps.

The second crucial step happened when a system of ramps was designed based on this principle. An architectural construction was developed simultaneously from all 4 sides of the pyramid. Thus, a detailed ramp system was available and a concrete model did exist.

Now a curtain has opened on a new world. For each level, the number of delivery lanes could be determined now. With this it was possible to calculate the number of workers and the construction time. This was the leap from only suggesting a principle to exact calculations.

The system of ramps was developed in a two months work in spring 2022. That was exactly 200 years later when Jean-François Champollion deciphered the Egyptian hieroglyphs. [27]

The following statements are essential for the construction process and differ from most of the other theories:

g2) The ramps are placed on the steps of the inner step pyramid.
The ramps lie on a horizontal base and have a stable hold. Attaching side flanks for additional stabilization is also possible.
Placing the ramps on the outer smooth casing is completely unstable and creates an enormous risk of avalanches.

g3) The transport is done by using rollers and without a sledge.
This requires that the blocks are already polished smooth when they are delivered and that the surfaces of the ramps are constantly being smoothed. Rolling friction requires only a sixth of the force of sliding friction. To reduce sliding friction, it is necessary to constantly pour water on wooden beams. This would require an immense transport of water and the barefoot workers would constantly slip on the wet ground.

g4) No oxen on the pyramid.
In ancient Egypt, oxen were used as draft animals. They may have been used to transport the blocks from the quarries or the preparation camp next to the pyramid. But on the pyramid, everything works according to a precise measure of time. There were the same laws as they do exist today on the assembly line of a large corporation. Everything had to be 100% under control. An ox with a sudden fear of heights would block an entire delivery lane.

g5) In order to avoid queuing at the corners, the ramp surfaces next to the corners are kept horizontal. There is no upward slope over a length of at least 12 m, so that the transport speed can be doubled in this segment.

g6) Measurements are possible at any time since the monument is constructed level by level. This is much more difficult when building in layers parallel to casing.

Finally, the question arises why the problem of building the pyramid could not be solved over the last 200 years.

27. https://en.wikipedia.org/wiki/Jean-Fran%C3%A7ois_Champollion

These are two possible reasons why the problem has not been solved for a long time:

e1) Archaeologists search in vain for writings, grave inscriptions, and traces in the sand. The guild of the pyramid builders was a high caste. They kept their knowledge strictly to themselves to preserve their power. Therefore, no writings would have been found in the destroyed library of Alexandria. That would be like finding the formula for Coca-Cola in the public library these days. [28]
The tomb paintings were mainly ritual texts that were intended to make the pharaoh's journey into the afterlife easier. They were not intended to record and transfer techniques.

e2) Engineers try to construct all sorts of machines.
There was neither iron nor steel at that time. The machines can only be driven by human power and must enable flexible mass transport.

The powerful and flexible lifting tools were the ingenious system of ramps. In addition, there was the collective cooperation of thousands of people, who acted like a herd of ants in a confined space. The population consisted mostly of farmers and labourers who did not possess any land and were seconded to compulsory labor. On the other hand, working on the pyramids during the Nile flood provided a varied experience with additional income. Labourers were proud to work on this monument. They wanted to make their contribution to this eternity machine. After his earthly death, the pharaoh was to ascend to the gods via this machine. From there he was supposed to tame fate. Not only the pharaoh, but all people in the state should receive eternal life. Moreover, catastrophes in the state should be kept away.

Stone was considered a symbol of eternity. That's why it is the only building material for the pyramid. The monument should be built for eternity. While only the wealthy could afford stone, dried brick was readily available and cheap. However, this weathered over the course of 200 years, especially under the influence of moisture.

According to the writings of Diodorus, Cheops was not buried in the pyramid. This agrees with the finding that the King's Chamber was not painted. The pharaoh alone had the absolute power in the state. This was very unpleasant for the lower ranked nobles. That is why the pharaoh had the fear that these could destroy his corpse and therefore, prevent his eternal life. This made the pharaoh change his mind several times in his long life and finally, had himself buried in a secret place.

Volz thinks that an alternative to the construction method only became possible with the invention of the steam engine in the 18th century. This was the first possibility to replace human power with another technique because animal power could not be used on the pyramid. [29]

It was the interaction of intuition, pragmatism, and mathematical modelling, what led to a simple approach and solution to the problem.

Only elementary arithmetic and simple geometric properties are used in this work to derive the results. The results could be obtained without a computer, however with a certain amount of manual arithmetic work. The construction planning could be done without large-scale paper. For this you need 4 walls with a width of 2 m and a height of 1.5 m. Lines are drawn on it and cords are attached with pins to indicate the course of the ramps. That was probably the architectural method of 5,000 years ago.

Due to the elementary approach, this work is accessible to a wide audience and can be understood by high school students.

28. https://en.wikipedia.org/wiki/Coca-Cola
29. https://en.wikipedia.org/wiki/Steam_engine

2. Table of Contents